# Integrated Optimal Controller for Polyvalent Heating/Cooling Systems with Minimal Cost and Energy Usage

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**Abstract:** In this paper the concept of an integrated optimal controller for polyvalent building heating and cooling systems is presented. The controller is named energy optimizer and employs control allocation for minimizing a cost function involving simplified models of several heating/cooling sources while delivering demanded heating/cooling energy for a given building. The cost function may be defined acc. to different objectives, for example to minimize energy usage and energy costs or to maximize the usage of renewable energy sources. Simplified simulation models for several components of a heating/cooling system (i.e. heat sources and sinks, energy diverter and mixer, storage, etc.) were derived and implemented in a simulation environment as well as in an automation station from the company SAUTER [1]. The concept of the energy optimizer was validated using Model-in-the-loop (MIL) and Hardware-in-the-loop (HIL), a detailed plant model was built using CARNOT (2013).

*Keywords:* Building automation, control allocation, optimal building control, control of energy efficient buildings, smart buildings

## 1. INTRODUCTION

The use of renewable energy sources in buildings has increased during past years, especially due to its ecological and possible financial benefits. However, due to its time-varying nature (for example solar irradiation), a polyvalent heating/cooling system design is required. Furthermore, each heating/cooling system is usually not operated the best way possible (with respect to energy saving) leading to higher energy consumption and sometimes to drawbacks regarding the indoor thermal comfort in the building. The energy optimizer (see Fig. 3) proposed in this paper should address these problems by: i) reducing the energy use and energy cost for heating/cooling energy supply; ii) delivering exactly required heating/cooling energy for a given building; iii) reducing the effort for comissioning of the building control system.

## 2. OPTIMAL CONTROL WITH CONTROL ALLOCATION (CA)

Optimal control deals with the task/challenge of taking the best decision for each instant of the system operation, i.e., finding the best solution in a determined sense. The best solution is a sequence of optimal decisions (control inputs) $\mathbf{u}_0^*, \mathbf{u}_1^*, ..., \mathbf{u}_{N-1}^*$ which minimizes a given cost/objective function $J$, also called performance (quality) index while satisfying the constraints. The cost function to be minimized may contain the time $t$, the control inputs $\mathbf{u}$, the states $\mathbf{x}$ or a combination of all, depending on the optimization objective to be achieved.

The control allocation (CA) deals with over-actuated systems, i.e., systems with more control inputs $\mathbf{u} \in \mathbb{R}^m$ than system outputs $\mathbf{y} \in \mathbb{R}^p, m > p$. Additional criteria may be added to benefit from these degrees of freedom, like for example, choosing the control inputs in a way that the energy consumption be minimized while still fulfilling the desired output $\mathbf{y}_{des}$. This method has been successfully employed for flight control systems (Holzapfel (2004); Filippi (2009)) as well as for vehicle dynamics control (Muffato (2009); Krueger (2007)).

The solution of a control allocation problem is performed by formulating it as a mathematical static optimization (minimization) problem, where the function to be minimized involves the control inputs $\mathbf{u}$ and may have equations representing the constraints to the system.

$$\begin{aligned} \min\ & J(\mathbf{u}) \\ \text{s.t. } & g_i(\mathbf{u}) = 0, \quad i = 1, \ldots, q \\ & h_j(\mathbf{u}) \le 0, \quad j = 1, \ldots, r, \end{aligned} \tag{1}$$

[1] `http://www.sauter-controls.com`

There are different methods used to solve optimal control problems, the most of them are based on two mathematical methods: The Bellman Principle of Optimality and the Pontryagin Maximum Principle (Bubnicki (2005); Sobotka and Buss (2009)). Previous work (Kelman and Borrelli (2011); Sturzenegger et al. (2014)) used model predictive control (MPC) for achieving optimal HVAC and building control. For using any of these methods, the dynamics of the plant must be formulated, for example as a state space model as in (2).

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \tag{2}$$

In this paper an optimal control method disconsidering the dynamic of the plant ($\dot{\mathbf{x}} = 0$ in (2)) is proposed. It uses CA with a simplified static simulation model (see section 3). We are interested in optimizing the usage of the heating/cooling sources during their steady-state operation, which is the longest phase and thus the most important during their operation. The transient phases may be disconsidered from the control problem due to their very short duration. Further advantage of disconsidering dynamics is that computation time is kept low which is positive regarding real time operation in an automation station (see section 6). The CA problem is solved at each discrete sample time and the optimal control input ($\mathbf{u}$) is applied to the plant/system. A drawback of this approach is that components with slow dynamics, for example a heat storage, cannot be considered dynamically during the optimization. The CA optimization problem is modelled with hard constraints. This has an advantage compared to the modelling with soft constraints, because there is no need for solving a multi-objective optimization problem and the effort for adjusting the weighting matrices for the different objectives is no longer required. This works very well, as long as the optimization problem is feasible, as in Fig. 1. However, in some moments of the system operation there will be unfeasible optimization problems stated and to deal with that the energy optimizer automatically switches between hard constraints and soft constraints, as shown in Fig. 2.

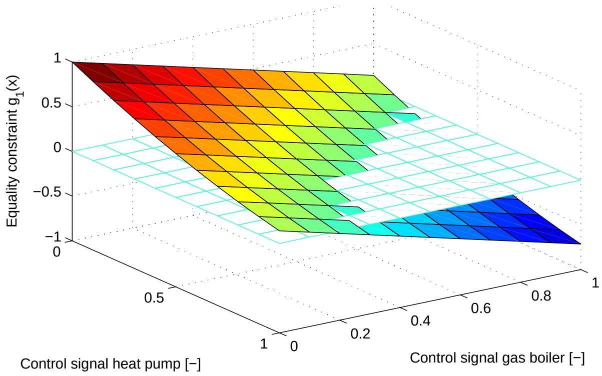


Fig. 1. Optimization for CA, the feasible region is not empty (the coloured surface which represents the equality constraint $g_1(\mathbf{u})$ as in section 2.2 meets the zero surface at several points). The control signals are normalized.

### 2.1 Cost function

The cost function / objective function for the energy optimizer with hard constraints is written:

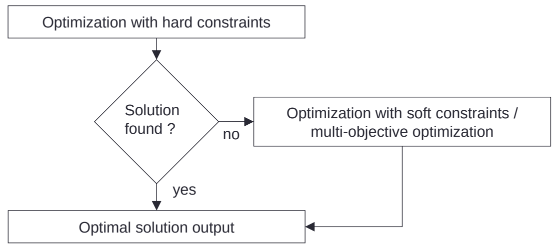


Fig. 2. Optimization switch from hard to soft constraints

$$J(\mathbf{u}) = \sum_{k=1}^{m} \dot{Q}_{\max,k}\, u_k\, w_k \tag{3}$$

with $m$ the number of heat sources, $\dot{Q}_{\max,k}$ the maximum amount of heat power possible to be produced by the $k^{th}$ heat source, $u_k$ the normalised value of the $k^{th}$ actuating variable, $w_k$ a weighting function for the $k^{th}$ heat source depending on the optimization goals (i.e. costs, primary energy, CO2 emissions, amount of renewable energy used). The cost function for the energy optimizer with soft constraints (multi-objective optimization) is written:

$$J_{soft}(\mathbf{u}) = J(\mathbf{u})RJ(\mathbf{u}) + \mathbf{g}(\mathbf{u})'Q\mathbf{g}(\mathbf{u}) \tag{4}$$

with R a scalar weighting factor, Q $\in \mathbb{R}^{q\times q}$ a weighting matrix and $\mathbf{g}(\mathbf{u}) \in \mathbb{R}^q$ a vector containing the equality constraints.

### 2.2 Constraints

The equality constraints for the CA problem as in (1) are used for achieving the desired set points chosen by the user of the energy optimizer. The set points may be a temperature or a mass flow at any point of the heating/cooling system or a combination of different states. The equality constraint for the CA problem used for controlling the total amount of heat power produced by the heating/cooling system is presented in (5).

$$g_1(\mathbf{u}) = \dot{Q}_{\text{is}}(\mathbf{u}) - \dot{Q}_{\text{setpoint}} \tag{5}$$

with $\dot{Q}_{\text{is}}(\mathbf{u})$ the total heat power produced. The inequality constraints $\mathbf{h}(\mathbf{u})$ for the CA problem are formulated based on the limits given for the normalised control inputs as in (6) and based on the limits given for the states as in (7).

$$\mathbf{0} \leq \mathbf{u} \leq \mathbf{1} \tag{6}$$

$$\mathbf{x}(\mathbf{u}) \in \Omega \subset \mathbb{R}^r \tag{7}$$

Constraints need to be scaled in order that the numerical optimization problem is well-posed. An example of how this may be performed for the equality constraint considering the total amount of heat power produced is shown in (8).

$$g_1(\mathbf{u}) = \frac{\dot{Q}_{\text{is}}(\mathbf{u}) - \dot{Q}_{\text{setpoint}}}{\dot{Q}_{\text{setpoint}}} \tag{8}$$

### 2.3 Calculation of derivatives

During MIL and HIL validation of the energy optimizer, it could be observed that a decisive step for the CA optimization to achieve a good convergence is the calculation of the gradients of the cost function as well as of the inequality and equality constraints. An algorithm has been developed for calculating gradients based on finite differences acc. to

Laporte and Tallec (2003). Furthermore, using a first-order Taylor series expansion, the directional derivative of the function $j(\mathbf{z})$ at a point $\mathbf{z}_0$ is simply identified with the difference between two of its values divided by $\varepsilon$:

$$grad(j(\mathbf{z}_0))_i = \frac{\partial j(\mathbf{z}_0)}{\partial z_i} \approx \frac{j(\mathbf{z}_0 + \varepsilon e_i) - j(\mathbf{z}_0)}{\varepsilon} \qquad (9)$$

with $\mathbf{z}_0 \in \mathbb{R}^r$, $i = 1, \cdots, r$, a sufficiently small chosen parameter $\varepsilon$ to avoid linearization and round-off errors and $(e_i)_{i=1,\cdots,r}$ denotes the canonical basis of $\mathbb{R}^r$.

### *2.4 Equipment controller*

The energy optimizer is designed for delivering demanded heating/cooling energy for a given building. This amount of energy is mainly based on the outdoor temperature, which changes slowly during a day (slow dynamics). However, controlling of HVAC components involves fast dynamics, for example the outlet temperature of the heating systems. In order to decouple both slow and fast dynamics, the energy optimizer should be implemented together with an equipment controller, as proposed in Fig. 3.

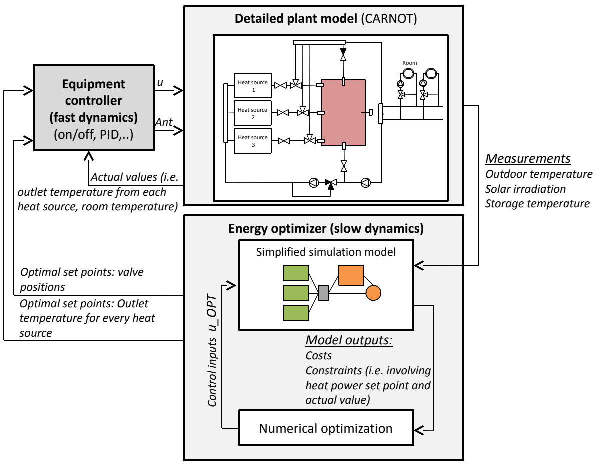


Fig. 3. Block diagram of the energy optimizer concept

## 3. SIMPLIFIED SIMULATION MODEL

Simplified simulation models for several components of a heating/cooling system (i.e. heat sources and sink, energy diverter and mixer, storage, etc.) were created for usage in the simulation environment as well as in an automation station (AS) modu525 from the company SAUTER (see section 6). Main purpose of the simplified models is to maintain the parametrization as simple as possible (e.g. using only datasheets of the component producers) and to estimate for the optimizer the total consumption or costs in dependence of the reference values of the building automation. Some of these models will be presented in this section.

### *3.1 Heat pump*

The efficiency $\eta$ is defined as ratio between the space (room) heating energy demand and the delivered energy for space heating which includes the heat losses. It is calculated based on the defined heating capacity:

$$\eta = \frac{\dot{Q}_{\mathrm{Gen}}}{\dot{Q}_{\mathrm{Drv}}} \qquad (10)$$

with $\dot{Q}_{\mathrm{Gen}}$ the heat energy produced by the generator and $\dot{Q}_{\mathrm{Drv}}$ the driving energy of the generator. The efficiency of the generator is defined as linear characteristic:

$$\eta_{\mathrm{Gen}} = u_{\mathrm{aCOP}}(t) \cdot aCOP + u_{\mathrm{bCOP}}(t) \cdot bCOP + cCOP \qquad (11)$$

where $u_{\mathrm{bCOP}}(t)$ and $u_{\mathrm{bCOP}}(t)$ are arbitrary values such as temperatures or capacities, which can be choosed dependent on the type of generator. Alternately, the flow temperature of the generator can be used as signal $u_{\mathrm{bCOP}}(t)$ in order to determine the efficiency. In this case the supply temperature and the COP are coupled and can be calculated internally in the generator.

In the case of the heat pump the efficiency $\eta$ is called COP, $\dot{Q}_{\mathrm{Drv}}$ corresponds to the purchased power from the grid and $\dot{Q}_{\mathrm{Gen}}$ is the heating capacity.

The costs (tariff, primary energy factor, renewable primary energy share, greenhouse gas emission coefficient) are calculated according to the equation:

$$K_{\mathrm{out}} = \frac{K_{\mathrm{Drv}} \cdot \dot{Q}_{\mathrm{Drv}} + K_{\mathrm{MSrc}} \cdot \dot{Q}_{\mathrm{MSrc}}}{\dot{Q}_{\mathrm{Drv}} + \dot{Q}_{\mathrm{MSrc}}} \qquad (12)$$

$K_{\mathrm{out}}$ represents the heating power costs of the heating source, $K_{\mathrm{Drv}}$ the cost of the purchased auxiliary power (e.g electrical power to supply the pumps and heating boiler), $K_{\mathrm{MSrc}}$ the cost of the main heating source (for example heating oil, wood, electricity) and $\dot{Q}_{\mathrm{MSrc}}$ is the heating power delivered by the main heating source. The output heating power is modeled according to the equation:

$$\dot{Q}_{\mathrm{Gen}}(t) = \dot{m}_{\mathrm{HW}}(t) \cdot cp_{,\mathrm{W}} \cdot (T_{\mathrm{Supply}}(t) - T_{\mathrm{Ret}}(t)) \qquad (13)$$

where $\dot{m}_{\mathrm{HW}}$ is the heating water massflow, $T_{\mathrm{Supply}}$ and $T_{\mathrm{Ret}}$ are respectively the supply and return temperature and $cp_{,\mathrm{W}}$ is the specific water heat capacity. Thereby, the supply temperature can be calculated:

$$T_{\mathrm{Supply}}(t) = \frac{\dot{Q}_{\mathrm{Gen}}(t)}{\dot{m}_{\mathrm{HW}}(t) \cdot cp_{,\mathrm{W}}} + T_{\mathrm{Ret}}(t) \qquad (14)$$

The supply temperature and the generator capacity can be limited in the model to a maximum supply temperature.

### *3.2 Solar collector*

The efficiency of a solar collector is normally modeled dependent on the ambient temperature, the average temperature in the collector and the solar irradiation, using the equation:

$$\eta_{\mathrm{Gen}}(t) = \eta_0 - \frac{T_{\mathrm{Amb}}(t) - T_{\mathrm{Supply}}(t)}{\dot{Q}_{\mathrm{Sun}}(t)} \cdot K_0 \qquad (15)$$

$$\dot{Q}_{\mathrm{Gen}}(t) = \eta_{\mathrm{Gen}}(t) \cdot \dot{Q}_{\mathrm{Sun}}(t) \qquad (16)$$

with $K_0$ being the heat loss coefficient and $\eta_0$ the efficiency without heat losses. In the simulation model a linear approximation of the efficiency is used:

$$\eta_{\text{Gen}}(t) = (T_{\text{Amb}}(t) - T_{\text{Supply}}(t)) \cdot K_1 + K_2 \tag{17}$$

The efficiency can thus be calculated:

$$\eta_{\text{Gen}}(t) = \frac{\dot{Q}_{\text{Gen}}(t)}{P_{\text{El}}(t)} = \frac{T_{\text{Amb}}(t) - T_{\text{Supply}}(t)}{P_{\text{El}}(t)} \cdot K_1 + \frac{\dot{Q}_{\text{Sun}}(t)}{P_{\text{El}}(t)} \tag{18}$$

A linear approximation as used in the simplified model can adequately reproduce the behaviour of the static collector operation over a wide temperature range.

### *3.3 Buildings loads*

The effective space heating or cooling load can be calculated as a linear characteristic, which is defined by the parameters:

$$\dot{Q}_{\text{Dem}} = \dot{Q}_{\text{Dim}} \frac{T_{\text{R,Ref}} - T_{\text{Amb}}}{T_{\text{Amb,Dim}} - T_{\text{Amb,Lim}}} \tag{19}$$

$\dot{Q}_{\text{Dim}}$ is the heating requirement at the minimal design temperature $T_{\text{Amb,Dim}}$. $T_{\text{Amb,Lim}}$ is the limit ambient temperature for which a heating is not required anymore. The supply temperature is similarly calculated:

$$T_{\text{Supply}} = \frac{(T_{\text{Supply Dim}} - T_{\text{R,Ref}})}{\dot{Q}_{\text{Dim}} + T_{\text{R,Ref}}} \tag{20}$$

The ideal return temperature is:

$$\dot{Q}_{\text{Dem}} = (T_{\text{Supply}} - T_{\text{Ret}}) \cdot \dot{m}_{\text{W}} \cdot cp_{,\text{W}} \tag{21}$$

$$T_{\text{Ret}} = T_{\text{Supply}} - \frac{\dot{Q}_{\text{Dem}}}{\dot{m}_{\text{W}} \cdot cp_{,\text{W}}} \tag{22}$$

The required heating $\dot{Q}_{\text{Dem}}$ is the reference used for the optimization. To calculate the heating costs, heating power and supply temperatures, the ideal return temperature is used as boundary condition of the simulation model. In this way no algebraic loop is introduced in the simulation and the whole system can be considered statically.

## 4. CONTROL SYSTEM VALIDATION

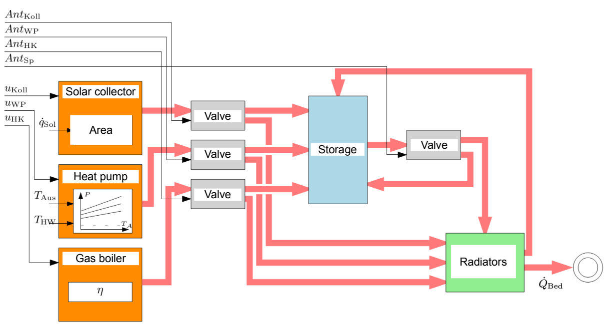


Fig. 4. Overview of the system employed for validating the energy optimizer

A polyvalent heating system was chosen for proofing the concept of the energy optimizer, as seen in Fig. 4. A building heating demand of 20 kW was calculated considering a room set-point temperature of 20 °C and an outside temperature of −8 °C. Heating is provided by 3 different systems: a solar collector, an air-to-water heat pump and a gas boiler. The heating systems are sized in order that each system can provide up to 50% of the total heating demand for the building. Furthermore, there is one storage for accumulating heat excess.

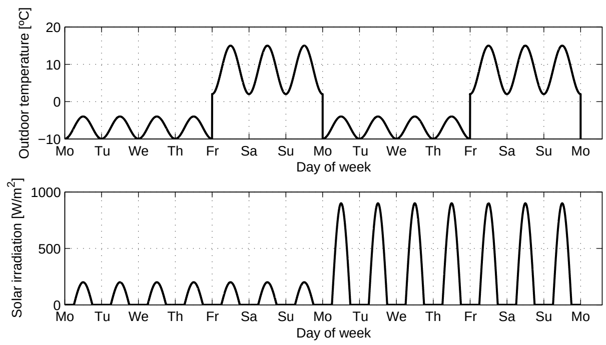


Fig. 5. Artificially generated weather data for a 14-day simulation

Weather data was artificially generated (see Fig. 5) in order to test the energy optimizer with a wide range of possible combinations of solar irradiation and outdoor temperature. Its behaviour for minimizing energy costs is expected as follows: i) at high solar irradiation, solar collector operation should be preferred; ii) at high outdoor temperature, heat pump operation should be preferred; iii) at low outdoor temperature and solar irradiation, gas boiler operation should be preferred.

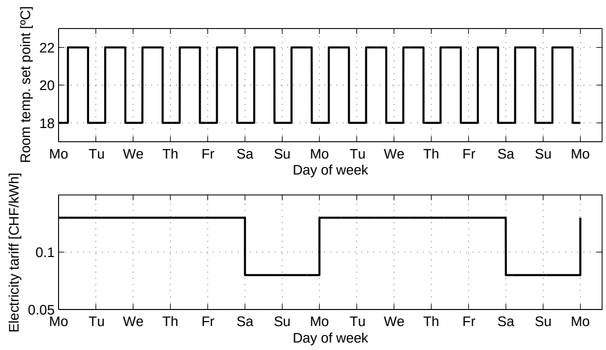


Fig. 6. Room temperature set point and electricity tariff for a 14-day simulation

Room temperature set point was chosen for a typical office room, the set point changes daily at 6 am and at 7 pm. Variable electricity tariff is becoming increasingly important in the context of intelligent electricity grids (Smart Grid) and the energy optimizer is capable of considering the current electricity tariff value (future costs may not be considered) in the control inputs calculation. For the simulations, tariff data obtained from a swiss provider for industrial consumers was used, as shown in Fig. 6.

### *4.1 Standard controller*

A standard controller for the system presented in Fig. 4 was designed for control performance comparison with the energy optimizer. The standard controller is based on well-established controllers for heating systems and consists of: i) an on/off solar collector controller which controls the collector pump based on the fluid outlet temperature and the solar irradiation. ii) an on/off heat pump controller based on the return flow temperature. iii) a gas boiler PI controller which controls the fluid outlet temperature by modulating the gas flow. The set point for the output temperature is calculated using a heating curve based on the outdoor temperature.

### *4.2 Energy optimizer*

The energy optimizer uses the simplified simulation model (as in section 3) from the system presented in Fig. 4 for performing the CA.

### *4.3 Energy optimizer with perfect plant model (PPM)*

The energy optimizer with the perfect plant model (PPM) uses a detailed plant model built using CARNOT (CARNOT (2013)) from the system presented in Fig. 4 for performing the CA. The results obtained are reference simulation results, which are only possible in real application if the perfect model of the plant would be available for the simulations. Differences between the energy optimizer and the energy optimizer (PPM) are equivalent to model-plant-mismatch (MPM) in real model based control applications.

## 5. SIMULATION RESULTS

In this section, the results from the 14-day simulations defined in section 4 are presented. The control performance is evaluated using the ISE criterion for the room temperature as in (23) and using the out of comfort time, i.e. an indicator for the number of hours the room temperature is outside of a given set point comfort zone.

$$\mathrm{ISE} = \int_0^\infty e^2(t)\,\mathrm{d}t \qquad (23)$$

with $e(t) = T_{\mathrm{room,set}} - T_{\mathrm{room,actual}}$. The room temperature set point tracking during day 2 of the 14-day simulation with different controllers is presented in Fig. 7. The small remaining tracking error from the control with the energy optimizer may be eliminated by reducing the dead zone of the room temperature controller and the tolerances in

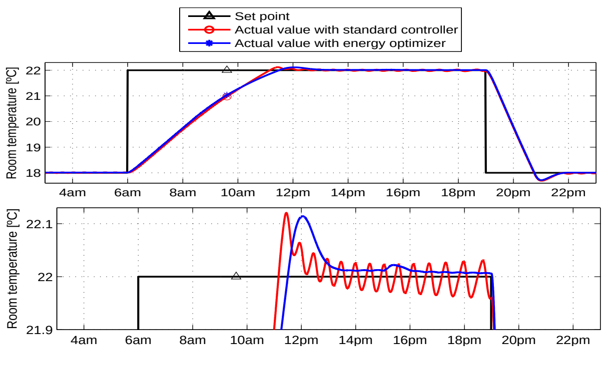


Fig. 7. Set point tracking comparison during day 2 of a 14-day simulation

the numerical optimization problem. The worst control performance is achieved with the standard controller. Furthermore, the continuous oscillation observed in the simulation would probably diminish lifetime of the control actuators involved in the real process. Comfort issues may be observed with both controllers as the set point changes from 18 to 22 °C, since prediction is not used in the control law.

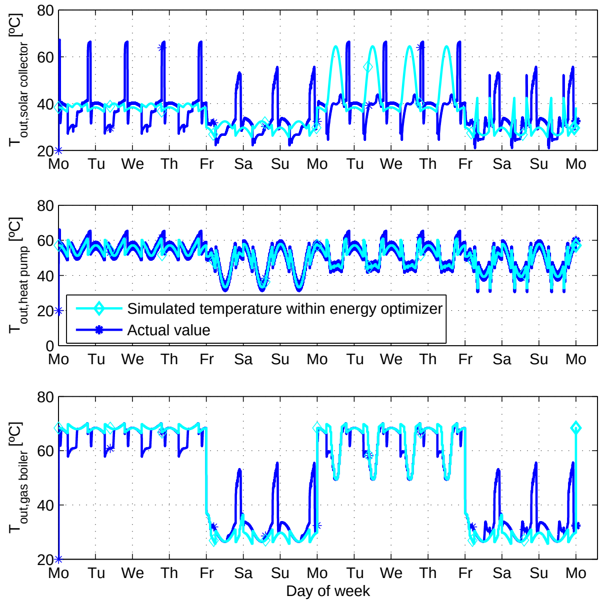


Fig. 8. Outlet temperature from several heat sources during a 14-day simulation

The energy optimizer computes the optimal outlet temperature for each heat source and these are forwarded as set points for the equipment controller, as seen in Fig. 8. The output of the real heat sources (obtained with the detailed simulation using CARNOT) is called actual value. Differences between simulated value and actual value exist and are caused by model-plant-mismatch (MPM). The impact

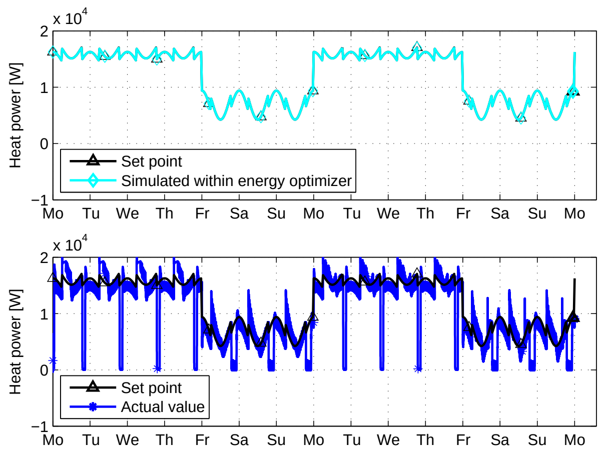


Fig. 9. Heat power set point and simulated value (top), set point and actual value (bottom)

of MPM may be seen very well for the solar collector: in the simulations performed within the energy optimizer, the solar collector achieves higher outlet temperatures than the actual value and with the actual value not enough heating power is produced, as seen in Fig. 9. Although in the simulations performed within the energy optimizer the heat power set point is achieved with maximum 0.4 % relative error (top of Fig. 9), the heat power set point is not achieved for several times (bottom of Fig. 9) using the real heat sources.

Table 1. Control performance comparison of a 14-day simulation

| **Plant model for performing the optimization** | **Equipment controller** | **ISE [%]** | **Cost [%]** |
|---|---|---|---|
| none (standard controller) | yes | 0 | 0 |
| simplified | yes | +16.7 | -5.8 |
| perfect | no | -1.9 | -8.0 |

A control performance comparison using different controllers is presented in Table 1. The best control performance is achieved with the energy optimizer with perfect plant model (PPM), with cost reduction and better comfort. For the energy optimizer – with simplified model – there is a cost reduction of 5.8% but the comfort diminishes 16.7% compared to the standard controller. These results highlight the need for very exact models, in order that control performance may be increased when using the energy optimizer in real applications.

## 6. HARDWARE IMPLEMENTATION, PRELIMINARY RESULTS

The modu525 (EY-AS525) is a SAUTER automation station designed for control, monitoring and optimisation of technical installations (SAUTER (2014)). The concept of the energy optimizer as described in this paper was successfully implemented and tested in a modu525 prototype. The nonlinear optimization problem is solved using NLopt (Johnson (2014); Kraft (1988)) embedded in modu525 to solve the CA problem. Several tests were performed and a solution for the CA problem could be found at each iteration of the energy optimizer. Further validation steps will be performed involving real HVAC equipment.

## 7. CONCLUSION

In this paper the concept of an energy optimizer was presented. The optimal controller is based on control allocation (CA) and has some advantages compared to a conventional controller. Additional objectives (for example to minimize energy use and energy cost or to maximize the use of renewable energy sources) may be considered and due to its feedforward characteristic the control inputs are calculated even before a tracking error exists, so that trajectory following is better/faster. The energy optimizer is capable of considering the current electricity tariff and room temperature set point in the optimal control inputs calculation, which is of importance in the context of intelligent electricity grids (Smart Grid).

Validation was performed using MIL and HIL, energy savings of up to 8% and comfort (ISE) increase of up to 1.9% could be achieved with the energy optimizer and the perfect plant model compared to a standard controller. Energy savings could be even higher depending on the plant configuration and on the standard controller parameters. However, since it is a model based controller, no energy saving and control robustness is guaranteed in the presence of parameter uncertainty or unmodeled dynamics.

Further investigation could be extending the controller capabilities to automatically detect model-plant-mismatch (MPM) and minimize it by adjusting the parameters of the simplified simulation model, thus keeping the performance of the model-based controller optimal. Another investigation could be comparing the presented controller to a rule-based controller (e.g. implementation effort and energy savings).

## 8. ACKNOWLEDGEMENTS

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